Surname:	**First Name:**

Current School:

Shrewsbury School

SIXTH FORM ENTRANCE EXAMINATION

2024 ENTRY

MATHEMATICS

(1 Hour 15 Minutes)

Instructions to candidates

Answer all questions, writing your answers in the spaces provided.

The number of marks for each question is shown in square brackets: [ ].

Section A contains questions of a GCSE nature. Attempt this section first, but do not spend too long on any particular question.

Section B is intended to be considerably more difficult, and is targeted at candidates who are either aiming for an academic scholarship on the strength of their mathematics or hoping to take Further Mathematics at A-Level.

You are expected to use a calculator in this examination.

Relevant working must be shown in order to gain high marks.

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

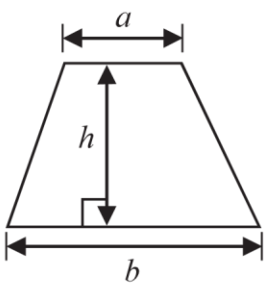
Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

The quadratic equation

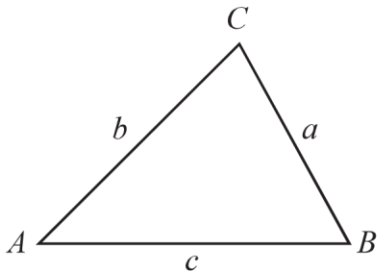
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \frac{1}{2}(a+b)h$



Trigonometry



In any triangle ABC

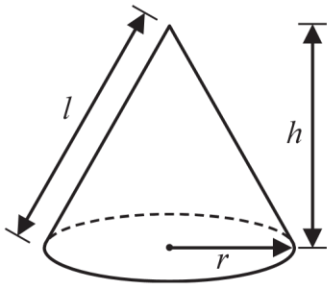
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

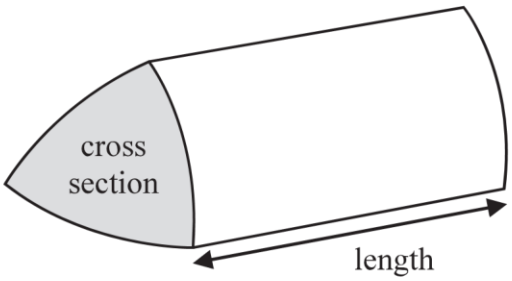
Area of triangle $= \frac{1}{2}ab\sin C$

Volume of cone $= \frac{1}{3}\pi r^2 h$

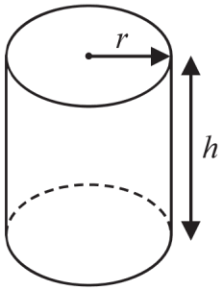
Curved surface area of cone $= \pi rl$



Volume of prism
= area of cross section × length

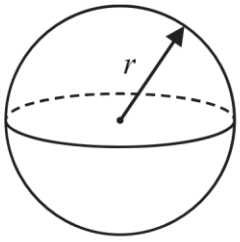


Volume of cylinder $= \pi r^2 h$
Curved surface area of cylinder $= 2\pi rh$



Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Section A (60 marks)

Answer all questions in this section.

1) Expand and simplify the following expressions:

a) $7(3t + 2u) - 4(5t - 6u)$

_____ [2]

b) $(5x - 2)(x + 4)$

_____ [2]

2) Factorise the following expressions fully:

a) $10ab^2 + 25a^2b$

_____ [2]

b) $n^2 + 11n - 26$

_____ [2]

3) You must **not** use a calculator in this question. Full working **must** be shown.

a) Evaluate these, giving your answers as **mixed numbers** in their simplest form:

i) $4\frac{2}{7} - 2\frac{4}{5}$

_____ [3]

ii) $4\frac{2}{7} \times 2\frac{4}{5}$

_____ [3]

b) Simplify the following:

$$(6y^{10}) \div (2y^{2})$$

_____ [2]

4) a) Scooby Doo lost 20% of his spaghetti plants in a particularly violent storm. If he had 35 plants before the storm, how many of his plants remained afterwards?

_____ [2]

b) A toy shop is having a Christmas sale in which the prices of all Lego sets are reduced by 45%. The Lion Knight's Lego Castle is on sale for only £189.75.

What is the normal selling price of this Lego set?

_____ [2]

c) In 1989, a fraudster was sentenced to 141,078 years in prison. She was released in 1993 after serving only 4 years of her prison sentence.

What was the percentage decrease in the length of the fraudster's prison sentence? Give your answer to **6** significant figures.

_____ [2]

5) a) A ladder of length 6 metres makes an angle of 75° with horizontal ground and rests against a vertical wall. Calculate how high the top of the ladder reaches up the wall, giving your answer to the nearest centimetre.

_____ [3]

b) The ladder is repositioned so that it reaches only 5 metres up the wall. Calculate how far the foot of the ladder is from the wall, giving your answer to the nearest centimetre.

_____ [3]

c) Calculate how much the angle between the ladder and ground has reduced.

_____ [3]

6) The volume V of an egg (measured in cm^3) is directly proportional to the **cube** of its length x (measured in cm).

When $x = 5, V = 50$.

a) Find a formula for V in terms of x.

_____ [3]

b) Calculate the length of an egg whose volume is 110 cm^3. Give your answer to 3 significant figures.

_____ [3]

7) Rearrange the following formulae to make *x* the subject:

a) $y = 4x + 9$

_____ [2]

b) $y = \frac{3x^2 - 1}{4}$

_____ [3]

c) $v = \frac{6t}{\sqrt{x}}$

_____ [3]

d) $p = \frac{3 - 2x}{x + 5}$

_____ [3]

8) Solve the following equations:

a) $7x - 2 = 3x + 16$

_____ [2]

b) $\frac{6}{2\sqrt{x}-3} = \frac{2}{5}$

_____ [3]

c) $3x^2 - 13x + 14 = 0$

_____ [3]

d) $x^2 + \frac{68}{x^2} = 21$

_____ [4]

Spare page for Section A working.

Please turn over for Section B.

Section B (20 marks)

This section is intended to be considerably more difficult, and is targeted at candidates who are either aiming for an academic scholarship on the strength of their mathematics or hoping to take Further Mathematics at A-Level.

Only attempt these questions if you have done as much of Section A as you can.

B1) a) Expand and simplify $(n-4)(2n+11)$.

_____ [1]

b) Hence, or otherwise, determine all integer values of *n* for which $\frac{2n^2+3n+5}{n-4}$ is also an integer.

(An *integer* is a whole number; it can be positive, negative, or zero.)

_____ [5]

B2) a) Eric flips three fair coins together. Calculate the probability that at least one of the coins lands on heads.

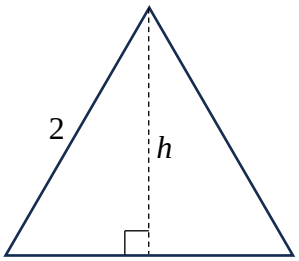
_____ [2]

b) Eric now flips k biased coins together n times in a row. Each coin has probability p of landing on heads. Eric wins a prize if **either** at least one coin lands on heads every time the k coins are flipped, **or** all the coins land on tails every time. Find an expression in terms of k, n and p for the probability that Eric wins a prize.

_____ [4]

Please turn over for the final question.

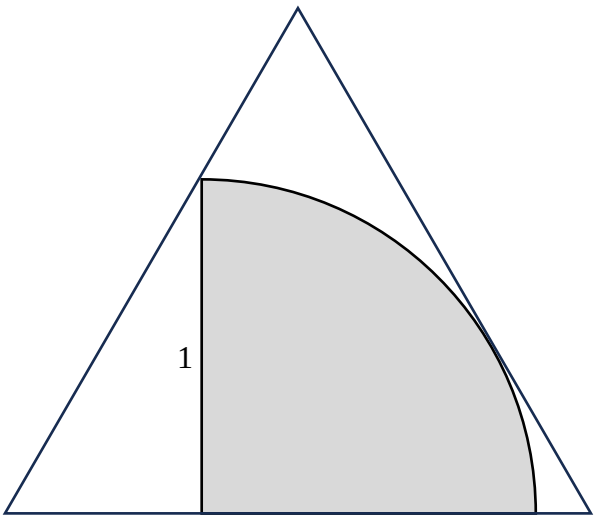
B3) The diagram below shows an equilateral triangle with sides of length 2 units and perpendicular height h.



a) Calculate the exact value of h.

_____ [1]

The diagram below shows a **different** equilateral triangle in which a quarter-circle of radius 1 unit is inscribed. The quarter-circle touches the left and right edges of the triangle, and its base lies along the bottom edge of the triangle.



b) Using similar triangles, or otherwise, find an exact expression for the proportion of the triangle that is occupied by the quarter-circle, giving your answer in terms of π.

_____ [7]

+++++ END +++++

Spare page for Section B working.